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Comments On the Confinement from Dilaton-Gluon Coupling in QCD

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Abstract

In this talk, I report on a work done in collaboration with R. Markazi and E.H. Saidi [1].

1 Introduction

Confinement in gauge theories provides one of the most challenging problems in theoretical physics. Various quark confinement models rely on flux tube picture. The latter emerges through the condensation of magnetic monopoles and explain the linear rising potential between color sources. However, a deep understanding of confinement mechanism is still lacking. Recently it has been shown in [2] that a string inspired coupling of a dilaton ϕ to the 4d $SU(N_c)$ gauge fields yields a phenomenologically interesting interquark potential $V(r)$ with a confining term. Extension of gauge field theories by inclusion of dilatonic degrees of freedom has gained considerable interest. Particularly, Dilatonic Maxwell and Yang Mills theories which, under some assumptions, possess stable, finite energy solutions [3].

2 Description of Dick Model

The Dick interquark potential [2] was obtained as follow: First start from the following model for the dilaton gluon coupling $G(\phi)$:

$$L(\phi, A) = -\frac{1}{4G(\phi)} F_{\mu\nu}^a F_a^{\mu\nu} - \frac{1}{2}(\partial_\mu \phi)^2 + W(\phi) + J_\mu^a A_a^\mu \quad (1)$$

Then construct $G(\phi)$ under the requirement that the Coulomb problem still possesses analytical solutions. The coupling $G(\phi)$ and the potential $W(\phi)$ that emerged are:

$$G(\phi) = \text{const.} + \frac{f^2}{\phi^2}, \quad W(\phi) = \frac{1}{2}m^2\phi^2 \quad (2)$$

where f is a scale parameter characterizing the strength of the scalar-gluon coupling and m represents the dilaton mass.

Next, consider the equations of motion of the fields A_μ and ϕ and solve them for static point like color source described by the current density $J_a^\mu = \rho_a \eta^{\mu 0}$. After some straightforward algebra, Dick shows that the interquark potential $V_D(r)$ is given by (up to a color factor),

$$V_D(r) = \frac{\alpha_s}{r} - gf \sqrt{\frac{N_c}{2(N_c-1)}} \ln[\exp(2mr) - 1] \quad (3)$$

Eq.(3) is remarkable since for large values of r it leads to a linear confining potential $V_D(r) \sim 2gfm \sqrt{\frac{N_c}{2(N_c-1)}} r$.

This derivation provides a challenge to monopole condensations as a new quark confinement scenario. Therefore, it is justified to dedicate more efforts to the investigation of a more general effective coupling function $G(\phi)$ and to the phenomenological application of Dick potential $V_D(r)$ [5].

In this regard, we have shown in [1] that for a general dilaton-gluon coupling $G(\phi)$, the quark interaction potential $V(r)$ reads as:

$$V(r) = \int dr \frac{G[\phi(r)]}{r^2} \quad (4)$$

Such form of the potential is very attractive. On the one hand, it extends the usual Coulomb formula $V_c \sim 1/r$ which is recovered from (4) by taking $G = 1$. Moreover for $G \sim r^2$, which by the way corresponds to a coupling $G(\phi) \sim \phi^{-2}$, and $W(\phi) = \frac{m^2}{2} \phi^2$, $m \neq 0$, Eq.(4) yields Dick solution. On the other hand Eq.(4) may be also used to relate non perturbative effects such as QCD vacuum condensates in term of dilaton parameter (m, f) . Indeed, following for instance[4], one may extract interesting phenomenological informations on the dilaton-gluon coupling $G[\phi]$ by comparing Eq.(4) to the Bian-Huang-Shen's potential $V_{BHS}(r)$ namely:

$$V_{BHS}(r) \sim \frac{1}{r} - \sum_{n \geq 0} C_n r^n \quad (5)$$

where C_n 's are related to the quark and gluon vacuum condensates. In fact one can do better if one can put the coupling $G(\phi)$ in the form $G[\phi(r)]$. In this case one can predict the type of vacuum condensates of the $SU(N_c)$ gauge theory which contributes to the quark interaction potential.

Although, the derivation of the formula (4) for the interquark potential from Eq.(1) is by itself an important result, there remain however other steps to overcome before one can exploit (4). A crucial technical step is to determine for what type of couplings $G(\phi)$, one can solve the equation of motion of the scalar field ϕ :

$$[D_\mu, G^{-1}(\phi) F^{\mu\nu}] = J^\nu \quad (a) \quad , \quad \partial_\mu \partial^\mu \phi = \frac{\partial W}{\partial \phi} - \frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} \frac{\partial G^{-1}(\phi)}{\partial \phi} \quad (b) \quad (6)$$

In trying to explore (4), we have observed that the functional $G[\phi(r)]$, and then the potential $V(r)$ of (4) may be obtained from the following one dimensional lagrangian

$$L_D = \frac{1}{2} (y')^2 + r^2 W(y/r) + \frac{\alpha}{2r^2} G(y/r) \quad (7)$$

where $y = r\phi$, $y' = (\frac{dy}{dr})$ and $\alpha = \frac{g^2}{16\pi^2} \frac{N_c-1}{2N_c}$ and where g is the gluon coupling constant. Indeed, if we start from (6) and set $F_a^{0i} = -\frac{gC_a}{4\pi} \partial_i V$, the equations of motion take a simple form,

$$\frac{dV}{dr} = r^{-2} G[\phi] \quad (a) \quad , \quad \Delta\phi = \frac{\partial W}{\partial\phi} + \frac{\alpha}{r^4} \frac{\partial G(\phi)}{\partial\phi} \quad (b) \quad (8)$$

Eq.(8.b) can be interpreted as corresponding to a mechanical system with the action:

$$S = \int dr r^2 [\phi'^2 + W(\phi) + \frac{\alpha}{2r^4} G(\phi)] = \int dr [\frac{1}{2} (y')^2 + r^2 W(y/r) + \frac{\alpha}{2r^2} G(y/r)] \quad (9)$$

Consequently the coupling $G(\phi)$ of Eq.(1) appears as a part of interacting potential of 1d quantum field theory.

3 Genaralized Dick model

First of all observe that the lagrangian (7) including the Dick model (1) is a particular one dimensional field theory of lagrangian

$$L = \frac{1}{2} (y')^2 - U(y, r) \quad (10)$$

where $U(y, r)$ is a priori an arbitrary potential. Though simple, this theory is not easy to solve except in some special cases. A class of solvable models is given by potentials of the form :

$$U(y) = \lambda^2 y^{2(n+p)} + \gamma^2 y^{2(q-n)} + \delta y^k \quad (11)$$

where n, p, q and k are numbers and λ^2, γ^2 and δ are coupling constants scaling as $(length)^{-2}$. The next thing to note is that Eq.(11) has no explicit dependence in r and consequently the following identity usually holds :

$$y'^2 = U + c \quad (12)$$

where c is a constant. Actually Eq.(12) is just an integral of motion which may be solved under some assumptions. Indeed by making appropriate choices of the coupling λ as well as the integral constant c , one may linearise y' in Eq.(12) as follows :

$$y' = U_1 + U_2 \quad . \quad (13)$$

Once the linearisation in y' is achieved and the terms U_1 and U_2 are identified, we can show that the solutions of Eq.(12) are classified by the product $U_1 U_2$ and the ratio U_1/U_2 . In what follows we discuss briefly some interesting examples.

3.1 First case: Dick solution

This corresponds to take $U_1 = my$ and $U_2 = c_1 y^{-1}$. Putting back into Eq.(12) one gets the Dick solution [2] which yields to the potential of Eq.(3).

3.2 Second case: New solutions

In this case the mass term is related to the product $U_1 U_2$ as:

$$U_1 U_2 = \pm \frac{1}{2} m^2 y^2 \quad (14)$$

Eq.(14) cannot however determine U_1 and U_2 independently as in general the following realizations are all of them candidates,

$$U_1 = \lambda y^{n+p}, \quad U_2 = \gamma y^{q-n} \quad (15)$$

where the integers p and q are such that $p + q = 2$ and where $\lambda\gamma = \pm m^2$. A remarkable example corresponds to take $p = q = 1$. In this case we distinguish two solutions according to the sign of the product of $\lambda\gamma$. For $\lambda\gamma = +m^2$, the solution is

$$y(r) = \left[\frac{1}{\lambda} \tan\left(\frac{nmr}{\sqrt{2}} + \text{const.}\right) \right]^{\frac{1}{n}} \quad (16)$$

For $\lambda\gamma = -m^2$, we have:

$$y(r) = \left[-\frac{1}{\lambda} \tanh\left(\frac{nmr}{\sqrt{2}} + \text{const.}\right) \right]^{\frac{1}{n}} \quad (17)$$

Note that one can go beyond the above mentioned solutions which are just special cases of general models involving interactions classified according to the following constraint:

$$U_1 U_2 \sim y^k \quad (18)$$

with $k (= p + q)$ an integer. Indeed, besides $k = 0$ and $k = 2$ which lead respectively to Dick solution and to the solutions given by (16,17); for general values of k , one has to know moreover the ratio U_1/U_2 in order to work out solutions. For the example where

$$\begin{aligned} U_1 &= \lambda y \\ U_2 &= \gamma y^{k-1} \quad ; \quad k \text{ integer} \end{aligned} \quad (19)$$

one can check, after some straightforward algebra, that the solution

$$y_k(r) = [r\phi_D]^{\frac{2}{(2-k)}} \quad (20)$$

is just a generalization of Dick solution (case $k = 0$).

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